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ABSTRACT: The formation and disruption of binaries in a near-equilibrium star cluster is analyzed. The authors conclude that in certain star clusters the mean star density is high enough for the formation of large numbers of binaries through random encounters of single stars. Mechanism of "evaporation" from such clusters is proposed to explain the relatively large numbers of binaries outside clusters.

We know that gravitational systems cannot be in a state of complete, statistical equilibrium. However, the fact, established by V. A. Ambartsumyan [1], that the time of dissipation of a gravitational system considerably exceeds the time of relaxation leads to the possibility of almost equilibrium states of such a system. This work is dedicated to the question of the formation and disruption of binary stars in such almost equilibrium states in the galaxy and individual star clusters. /273

The main conclusions are as follows:

1. The time during which the equilibrium number of star pairs is formed due to approaches of three stars is comparable with the time required for their disruption due to random encounters with single stars.

2. The average evolution of a pair due to random approaches by surrounding stars depends on whether the pair is "strong" or "weak," i.e. whether the energy of its orbital movement is greater or less in absolute value than the mean energy of the approaching movement of stars approaching at a single degree of freedom. Strong pairs tend to decrease a, i.e. "become stronger," while weak pairs tend to increase a, i.e. to separate.

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[1] Numbers in the margin indicate pagination in the foreign text.

3. The time of formation of weak pairs is less than the time of relaxation, so that they can be formed in star systems in a statistically equilibrium quantity. The time of formation of strong pairs is greater than the relaxation time and increases approximately as $1/a^{3/2}$. Therefore, in star systems, pairs for which a is an order of magnitude less than $2a_0$ -- the boundary value between strong and weak pairs -- can form in almost equilibrium quantity.

4. Analysis of equilibrium dissociation of star paris in consideration of the gravitational interactions between components shows that the equilibrium state of the star systems should correspond to a unique "condensation" of the stellar gas, i.e. the formation of a single, close, multiple system.

Actually, this state is not achieved, since in correspondence with item 3 above the time of formation of very close pairs increases without limit as the length of the half axis decreases.

5. The abundance of binary stars in the neighborhood of the sun and the decreasing nature of the curve of distribution by lengths of half axes can be explained quantitatively by the assumption that these pairs were formed in galactic star clusters and "evaporated" from them.

This assumption agrees with the opinion recently stated by V. A. Ambartsumyan that the formation of stars occurs in stellar associations.

1. Kinetics of establishment of statistical equilibrium. The time during which a considerable share of the equilibrium number of binary stars is formed due to random approaches of three stars is comparable with the time of disruption of pairs by random encounters with surrounding stars, estimated /274
for wide Ambartsumyan pairs [2].

Let us analyze a stellar system in which the number of binary stars is much less than the number of single stars. Then, the formation of pairs by capture and disruption of pairs will occur in the overwhelming majority of cases under the influence of single stars. Captures and separations under the influence of other pairs can be ignored. The change in the number of pairs is determined approximately by the equation:

$$\frac{dn_{12}}{dt} = \alpha n^2 - \beta n n_{12}. \tag{1.1}$$

here n is the number of single stars per unit volume, while n_{12} is thc number of pairs within a certain interval of large half axis lengths a. We limit ourselves in (1.1) only to captures and separations, attempting no analysis of the detailed kinetics involved in gradual changes of half axes, since our purpose is only to produce a qualitative estimate of the probability of capture.

In the equilibrium state $dn_{12}/dt = 0$ and, consequently,

$$\frac{\alpha}{\beta} = \frac{n_{12}^0}{n_0^2}, \tag{1.2}$$

where the zero represents the equilibrium state. Since the share of binary stars is small, practically $n = n_0$. Considering this and substituting (1.2) into (1.1), we produce:

$$\frac{dn_{12}}{dt} = \beta n_0 (n_{12}^0 - n_{12}), \tag{1.3a}$$

from which

$$n_{12} = n_{12}^0 (1 - e^{-\beta n_0 t}) = n_{12}^0 (1 - e^{-t/t_0}). \tag{1.3b}$$

If the formation of a pair does not occur, then (1.3a) takes on the form:

$$\frac{dn_{12}}{dt} = -\beta n_0 n_{12}.$$

From this we see that $t_0 = 1/\beta n_0$ is the time constant for destruction of pairs by "shocks" of single stars. Formula (1.3b) shows that this is also the time of their formation in a quantity near the equilibrium quantity. Therefore, the study of the kinetics of the formation of pairs can be replaced by a study of the kinetics of their disruption.

In a star system in which movement is distributed without order, the average result of the approach of two stars is that the star with greater energy transmits energy to the star with lesser energy. Thus, in correspondence with the H-theorem, the energy tends to approach equilibrium.

Suppose stars of mass m_1 and m_2 have velocities v_1 and v_2 before the approach, v'_1 and v'_2 after the approach. If we represent the velocity of the center of inertia by v_c and the relative velocities before and after approach by v and v', the change in energy of the first star will be

$$\Delta E_1=\frac{1}{2}m_1(v_1'^2-v_1^2)=\frac{1}{2}m_1\left[\left(\frac{m_2}{m_1+m_2}\right)^2(v'^2-v^2)+\right.$$

$$\left.+2\frac{m_2}{m_1+m_2}(\mathbf{v}_c\mathbf{v}'-\mathbf{v}_c\mathbf{v})\right]=\frac{m_1m_2}{m_1+m_2}v_cv(\cos\varphi'-\cos\varphi),$$

where ϕ and ϕ' are the angles (v_cv) and (v_cv'). Let us construct, using v_c, v and v', a spherical triangle with dies $\phi_1\phi'$ and χ (scattering angle vv') and angle ψ between sides χ and ϕ. Then /275

$$\cos\varphi'=\cos\varphi\cos\chi+\sin\varphi\sin\chi\cos\psi$$

and therefore

$$\cos\varphi'-\cos\varphi=-2\cos\varphi\sin^2\frac{\chi}{2}+2\sin\varphi\cos\frac{\chi}{2}\sin\frac{\chi}{2}\cos\psi.$$

Since the values of ψ are equally probable, then on the average

$$\overline{\Delta E_1}=-2\frac{m_1m_2}{m_1+m_2}\mathbf{v}\mathbf{v}_c\sin^2\frac{\chi}{2}=-\frac{2m_1m_2}{(m_1+m_2)^2}\left[m_1v_1^2-m_2v_2^2+\right.$$
$$\left.+(m_2-m_1)\mathbf{v}_1\mathbf{v}_2\right]\sin^2\frac{\chi}{2}.$$

As we know

$$\sin^2\frac{\chi}{2}=\frac{1}{1+\frac{r^2v^4}{\gamma^2(m_1+m_2)^2}},$$

where r is the citing distance, γ is the gravity constant. Averaging with respect to all values of angle (v_1v_2) and considering that v also depends on this angle, we produce:

$$\overline{\Delta E_1}=\frac{\gamma m_1m_2(v_2^2-v_1^2)}{4rv_1v_2}\operatorname{arctg}\frac{\frac{4rv_1v_2}{\gamma(m_1+m_2)}}{1+\frac{r^2(v_2^2-v_1^2)^2}{\gamma^2(m_1+m_2)^2}}+$$
$$+\frac{\gamma^2m_1m_2(m_2-m_1)}{8r^2v_1v_2}\ln\frac{1+\frac{r^2(v_1+v_2)^4}{\gamma^2(m_1+m_2)^2}}{1+\frac{r^2(v_1-v_2)^4}{\gamma^2(m_1+m_2)^2}}. \qquad (1.4)$$

The limit expressions for $\overline{\Delta E}$ can be arbitrarily represented in a clearer form:

$$\overline{\Delta E}_1 = 4\frac{m_1 m_2}{(m_1+m_2)^2}\frac{E_2 - E_1}{1+\frac{r^2 v^4}{\gamma^2 (m_1+m_2)^2}}, \tag{1.4a}$$

which is correct either where $m_1 = m_2$ or where $r \approx 0$, or where $v_1 \approx 0$ or $v_2 \approx 0$.

We will consider a pair to be "strong" if the energy of relative movement $u = -\gamma m_1 m_2/2a$ is greater in absolute value than $\theta/2$, the mean energy per degree of freedom ("gravitational temperature" of the approaching movement of the stars $\theta = \overline{mv^2}/3$). We will consider a pair to be "weak" if $|u| < \theta/2$. The value of half axis for which $|u| = \theta/2$ will be represented by $2a_0$

$$2a_0 = \frac{\gamma m_1 m_2}{\theta} = \frac{3\gamma m_1 m_2}{\overline{mv^2}}. \tag{1.5}$$

Thus, the strong or weak nature of a pair is determined as a function of the "temperature" of the stellar medium in which it is located. The basis for these terms will be explained in the following.

Encounters of strong and weak pairs with other stars lead on the average to essentially different results. In the case of a weak pair ($a \gg 2a_0$) the energy of the relative (internal) movement is small in comparison with the energy of the oncoming star, equal on the average to $(3/2)\theta$, and in comparison with the same energy of movement of the center of inertia of the pair.

But then we can consider approximately that each star of the pair has on the average energy $< (3/2)\theta$ (or with equal mass, energy $(3/4)\theta$), and therefore, according to (1.4), the average result of the approach will be an increase in the energy of the pair, i.e. an increase in the size of the orbit. In the following, we will limit ourselves to rough estimates. Considering for simplicity that all of the stars being analyzed have identical mass m and assuming that on the average $|E_2 - E_1| \approx \theta$, we produce from (1.4) /276

$$\overline{\Delta E}_1 = \frac{\theta}{1+\frac{r^2 v^4}{4\gamma^2 m^2}}.$$

The pair will be broken up if the energy received by either component of the pair becomes equal to the coupling energy $\gamma m^2/2a$. This may occur as a result of one approach to a sighting distance such that $\overline{\Delta E_1} = \gamma m^2/2a$, from which

$$r^2 = \frac{4\gamma^2 m^2}{v^4}\left(\frac{2a\theta}{\gamma m^2} - 1\right) \approx \frac{8\gamma a\theta}{v^4}, \tag{1.6}$$

since for weak pairs $\gamma m^2/2a \ll \theta$. The average waiting time for such an approach will be

$$t_1 = \frac{1}{nv\pi r^2} = \frac{v^3}{8\pi\gamma n a\theta}.$$

Since $\theta = \frac{\overline{mv^2}}{3}$, then, ignoring the difference between the average velocity and the relative velocity, we produce:

$$t_1 = \frac{3}{8\pi}\frac{v}{\gamma m n a}. \tag{1.7}$$

In this calculation we assumed that $r \ll a$, otherwise the oncoming star, in place of simple transmission of energy to one component of the pair, will create a "tidal" type force which will have little effect where $r > a$. It is easy to see that inequality $r \ll a$ is fulfilled for weak pairs. Actually, according to (1.6), substituting $v^2 = 3\theta/m$,

$$\left(\frac{r}{a}\right)^2 = \frac{8\gamma\theta}{av^4} = \frac{8}{9}\frac{\gamma m^2}{a\theta} \ll 1.$$

If the disruption of the weak pair occurs due to many "weak" collisions, then in our approximation $|E_2 - E_1| \approx \theta$ it is sufficient to determine

$$\frac{dE}{dt} = \int_0^{r_m} \overline{\Delta E_1} nv\, 2\pi r dr = \pi n v\theta \int_0^{r_m} \frac{2r dr}{1 + \frac{r^2 v^4}{4\gamma^2 m^2}} = \pi n v\theta \frac{4\gamma m^2}{v^4} \ln\left(1 + \frac{r_m^2}{4\gamma^2 m^2}\right).$$

On the basis of the above, $r_m = a$. Here

$$\frac{r_m^2 v^4}{4\gamma^2 v^4} = \left(\frac{3}{2}\frac{a\theta}{\gamma m^2}\right)^2 \gg 1,$$

so that

$$\frac{dE}{dt}=\frac{8\pi\gamma^2m^2n\theta}{v^3}\ln\frac{av^2}{2\gamma m}.$$

Therefore, the time during which the energy of the pair increases by the value of the coupling energy $\gamma m^2/2a$, i.e. the time required for disruption of the pair, is equal to /277

$$t_1=\frac{v^3}{16\pi\gamma na\theta\ln\frac{av^2}{2\gamma m}}=\frac{3v}{16\pi\gamma mna\ln\frac{av^2}{2\gamma m}}. \tag{1.8}$$

More precise calculation of the mean value of the square of the change in energy over the relaxation time $\Delta E^2/\tau$, determined by the "coefficient of diffusion of energy into space" decreases this expression slightly, although even without this the expression is somewhat less than (1.7).

Consequently, disruption of weak pairs occurs primarily due to "weak" i.e. distant encounters, and the time required for disruption is inversely proportional to the length of the longer half axis. Obviously, where $a \gg 2a_0$, time t_1 is much less than the relaxation time t_r of the star system in which the pair is located. This follows from the fact that the former is the time during which the energy of the pair changes by the value of the coupling energy, while the latter is the time during which the energy changes by much larger quantity θ. This is especially true since t_1 is much less than the dissipation time of the system. Therefore, weak pairs can be formed in a star system in equilibrium quantity.

Let us go over to analysis of very strong pairs. In this case, the kinetic energy of the relative movement is greater than θ, i.e. on the average greater than the energy of an oncoming star until it approaches the binary star. Due to this, the average result of an approach will be transmission of energy to the oncoming star, i.e. a reduction in the size of the orbit of the pair. Thus, strong pairs, in case of random approaches, generally become stronger. However, formula (1.4) and this conclusion from it are true only for rather close approaches. The reason for this is that (1.4) was concluded for the approach of two free masses, while actually the component of the pair which is approaching the oncoming star is coupled to the other component of the pair. Therefore, (1.4) is true only for rather brief approaches, during which the duration of the effective interaction of the approaching stars is much less than the period of the orbit of the binary star. This condition will be fulfilled if

$$\frac{r}{v}<\frac{a}{v_0},$$

where v_0 is the orbital velocity. Since for very strong pairs $v_0 \approx \sqrt{\gamma m/a}$ is much greater than the average velocity of oncoming stars, $v \approx v_0$, and therefore the condition produced is equivalent to the previous condition $r_m \approx a$, used in concluding (1.6).

With great sighting distances and times of interaction, the approach will occur "adiabatically" in the sense that the energy taken from the oncoming star from the orbital movement will decrease rapidly. Therefore, it is sufficient to integrate up to $\mathbf{r} = a$. However, in this case in (1.4) $a^2 v_0^4/4\gamma^2 m^2 \approx 1$, while the energy transmitted $\overline{\Delta E_1} \approx E_1$, i.e. the energy of the strong pair is transmitted with a single approach to sighting distance $\leqslant$ a. In other words, during a single such approach, an essential "strengthening" of the pair can occur. The waiting time for such an event is

$$t_2 = \frac{1}{nv\pi a^2} \approx \frac{1}{nv_0\pi a^2} \approx \frac{1}{\pi n a^2}\sqrt{\frac{a}{\gamma m}} \infty a^{-3/2}. \tag{1.7}$$

Since where $a \geqslant 2a_0$ the time of formation of the pair is determined by formula (1.6), while where $a \leqslant 2a_0$ it is determined by formula (1.7), where $a \approx 2a_0$ both formulas should give the same order of magnitude for the /278
formation time: $t_2(2a_0) \approx t_1(2a_0)$.

Let us compare the time t_2 of formation of a strong pair with a large half axis a and time t_0 of formation of a pair with "critical" half axis $2a_0$. According to (1.7)

$$\frac{t_2}{t_0} \approx \left(\frac{2a_0}{a}\right)^{3/2}.$$

But for pairs with half axis $2a_0$ the disruption time, and consequently the time of formation according to the definition of a_0, is comparable with the relaxation time. If we accept the estimate of V. A. Ambartsumyan [1], according to which the time of dissipation is two orders of magnitude greater than the time of relaxation, during the time of dissipation of the star system, strong pairs will form for which the half axis is 10 to 20 times less than the critical value $2a_0$. Even stronger pairs will form in a quantity amounting to only a small fraction of the equilibrium quantity.

The conclusions produced in this paragraph contradicted the erroneous conclusion of Jeans [3], who stated that when a weak pair approaches a bypassing star the increase in orbital velocity will be accompanied by a decrease in the period (and consequently a decrease in the large half axis), while strong pairs, as a result of the decrease in orbital velocity, will show an increase in period (lengthening of large half axis). Obviously, this contradicts mechanics, as well as the nature of the equilibrium distribution function of pairs by periods which in the case of the "mean" period of Jeans has no maximum at all. The distribution function by half axis even has a minimum at this point.

2. Statistical equilibrium of binary stars. According to a general principle of statistical physics, the probability of location of a system in a certain interval of states is determined by the phase volume Z of this interval, calculated considering the "weight" energy factor $e^{-\epsilon/kT}$;

$$Z = \int e^{-\frac{\epsilon}{kT}} (dp)\,(dq),$$

where (dp) and (dq) are the products of the differentials of the impulses and coordinates of the system. Let us apply this principle to binary stars, which may be either in the "coupled" or "free" states corresponding to negative and positive energies, calculating the number of stars and binary stars per unit volume. For coupled pairs, the weighted phase volume is divided into factors related to the approaching movement of the center of inertia (Z_c) and the relative movement of the components of the pair (Z_{12}) so that the number of pairs of stars types 1 and 2 in a certain interval of states per unit volume will be

$$n_{12} \backsim Z_{12} Z_c$$

(~ represents proportionality). For free pairs we produce the product of the phase volumes of both components:

$$n_1 n_2 \backsim Z_1 Z_2,$$

from which

$$\frac{n_{12}}{n_1 n_2} = \frac{Z_{12} Z_c}{Z_1 Z_2}. \tag{2.1}$$

If the "gas" of stars can be considered ideal, then in calculating Z_1 and Z_2 we need not explicitly consider interactions between stars (the gravitational field is included only implicitly through the average kinetic energy of the stars or the "temperature" θ), i.e. approaches during which mutual potential energy is comparable with the average kinetic energy. /279

This condition leads to the fact that $Z_{12} \ll Z_1$ and Z_2 or $n_{12} \ll n_1$ and n_2. Then

$$Z_1 = (2\pi m_1 \theta)^{3/2}; \qquad Z_2 = (2\pi m_2 \theta)^{3/2}; \qquad Z_c = [2\pi (m_1 + m_2)\theta]^{3/2},$$

$$Z_{12} = \int e^{-\frac{u}{\theta}} dp_x dp_y dp_z dx dy dz.$$

Let us introduce the Delone variables [4] for relative movement, i.e. for movement of a body with mass $m_1 m_2/(m_1 + m_2)$ in the field of a center of attraction with mass $m_1 + m_2$:

$$L = \sqrt{\frac{m_1^2 m_2^2}{m_1 + m_2}\gamma a}; \quad G = L\sqrt{1-\varepsilon^2}; \; H = G\cos i = L\sqrt{1-\varepsilon^2}\cos i,$$

where ϵ is the eccentricity, i is the inclination of the orbit. The clear sense of these variables is: L is the moment of the quantity of movement in a circular orbit of radius a, G is the moment in an elliptical orbit with eccentricity ϵ, H is its projection on the direction separated. The canonical conjugate angular variables are: l, the mean anomaly; g, the distance of the periastron from the node, h is the length of the ascending node. Since $u = -\gamma m_1 m_2/2a$,

$$Z_{12} = \int e^{-\frac{u}{\theta}} dL\, dG\, dH\, dl\, dg\, dh = 8\pi^3 \int e^{-\frac{u}{\theta}} dL \int_0^L dG \int_{-G}^{+G} dH =$$

$$= 8\pi^3 \int e^{-\frac{u}{\theta}} L^2 dL = 4\pi^3 \gamma^{3/2} \frac{m_1^3 m_2^3}{(m_1+m_2)^{3/2}} \int_{a_1}^{a_2} e^{\frac{\gamma m_1 m_2}{2a\theta}} \sqrt{a}\, da.$$

Substituting the expressions for the statistical integrals Z into (2.1), we produce

$$\frac{n_{12}}{n_1 n_2} = \sqrt{2}\left(\frac{\pi\gamma m_1 m_2}{\theta}\right)^{3/2} \int_{a_1}^{a_2} e^{\frac{\gamma m_1 m_2}{2a\theta}} \sqrt{a}\, da = 4\,(\pi a_0)^{3/2} \int_{a_1}^{a_2} e^{\frac{a_0}{a}} \sqrt{a}\, da. \tag{2.2}$$

The differential equation for the distribution of pairs consisting of stars of these types by large half axes is

$$\frac{dn_{12}}{n_1 n_2} = \sqrt{2}\left(\frac{\pi\gamma m_1 m_2}{\theta}\right)^{3/2} e^{\frac{\gamma m_1 m_2}{2a\theta}} \sqrt{a}\, da = 4\,(\pi a_0)^{3/2} e^{\frac{a_0}{a}} \sqrt{a}\, da. \qquad (2.3)$$

We note that the distribution by periods, equivalent to (2.3), was produced by Jeans [3], although with an accuracy to an undefined factor. V. A. Ambartsumyan [2, 5] gave an approximate estimate of this factor.

Completely analogously we can find the distribution for typical trinary stars consisting of pairs (1,2) with half axis a plus a distant satellite (3) rotating about the pair in an orbit with half axis a'. In the interval

$$a_1 < a < a_2, \quad a_1' < a' < a_2'$$

we produce a number of trinary stars (per unit volume)

$$\frac{n_{123}}{n_1 n_2 n_3} = 4\,(\pi a_0)^{3/2} \int_{a_1}^{a_2} e^{\frac{a_0}{a}} \sqrt{a}\, da \cdot 4\,(\pi a_0')^{3/2} \int_{a_1'}^{a_2'} e^{\frac{a_0'}{a}} \sqrt{a}\, da, \qquad (2.4)$$ /280

where

$$a_0' = \frac{\gamma (m_1 + m_2) m_3}{2\theta}.$$

For typical quaternary stars, consisting of the two closest pairs, in the expression for $n_{1234}/n_1 n_2 n_3 n_4$ in the righthand side a cofactor of the same structure appears, and so forth for stars of higher degrees of multiplicity.

Let us note two properties of the equilibrium distribution (2.3):

1) The more massive stars in statistical equilibrium form paris in higher percentages (therefore, the luminance function for pairs should be somewhat different than for single stars). Also, the greater the mass of the stars, the greater the predominance of close pairs. Both of these conclusions are confirmed qualitatively by observations [6].

2) The distribution function first decreases as the half axis increases to $a = 2a_0$, then increases. This value $a = 2a_0$ is the boundary between pairs having a tendency to become stronger and pairs having a tendency to expand. We can see from the distribution curve that this involves a tendency to transition from statistically less probable states to statistically more probable states.

In the area of the sun $v = 2 \cdot 10^6$ cm/sec and for stars of solar mass $2a_0 = 10^{14}$ cm = 7 astronomical units, increasing for massive supergiants to hundreds of astronomical units. However, in star clusters, where $v \approx 10^5$ cm/sec and less, $2a_0$ increases to thousands of astronomical units even for stars of medium mass, i.e. the dipping portion of the curve encompasses almost all the observed range of large half axes.

In order to estimate the complete share of pairs, we must integrate (2.2) with respect to all values of a from $a_1 \approx 10^{11}$ cm for the closest spectroscopic binaries to the largest observed stars ($a_2 = 3 \cdot 10^{17}$ cm).

This integral can be calculated as the sum of integrals with respect to strong and weak pairs: $\int_{a_1}^{2a_0} + \int_{2a_0}^{a_2}$. Since $a_1 \ll a_0$, we can assume in the first integral $a = a_1 + \xi$, and expand the index $a_0/a = a_0/a_1(1 - \xi/a_1)$, replacing $\sqrt{a}$ by $\sqrt{a_1}$, which is proper due to the rapid decrease in the exponential function, and replace the upper limit of $2a_0$ by infinity for the same reason. Since $2a_0 \ll a_2$, we can replace the exponential factor by unity in the second integral, and replace the lower limit of $2a_0$ by zero. Then

$$\frac{n_{12}}{n_1 n_2} = 4 (\pi a_0)^{3/2} \left(\frac{a_1^{5/2}}{a_0} e^{\frac{a_0}{a_1}} + \frac{2}{3} a_2^{3/2} \right). \tag{2.5}$$

In the area of the sun for moderate stellar masses $m = 0.4 m_\odot$, the indicator $a_0/a = 10^2$. For massive stars and in star clusters where θ and v are considerably less, this indicator increases. Thus, we come to the following conclusion.

Statistical equilibrium under galactic conditions corresponds to the fact that practically all binary stars must be quite close. The colossal value of the exponential function, which cannot be compensated by the pre-exponential factors, shows that in statistical equilibrium with the "temperature" θ of the galaxy (and particularly at the "temperatures" of the star clusters) all of the stars should become binary, trinary and similar /281
aggregates, i.e. "condensation" of the star gas should occur. Here, the formulas which we have written cease to become quantitatively correct, since the single and multiple stars cannot be considered "atoms" and "molecules" of an ideal gas, but rather we must consider the interactions in the integrals for the translational degrees of freedom in (2.1) but, nevertheless, the qualitative conclusion remains correct.

This "condensation" cannot occur since, as we know, the time required for formation of strong pairs is very great, increasing without limit as the half axis length is decreased, and therefore becoming much greater than the time of dissipation of the system. To this, we must add cosmogonic factors (in the narrow sense of the term), for example evolution of individual stars.

For rather broad, weak pairs, equilibrium is established rather quickly, so that the conclusions based on our analysis of statistically equilibrium states can be applied to them. Ambartsumyan showed that in the area of the sun the number of broad pairs is many orders of magnitude greater than the equilibrium number corresponding to the conditions in the area of the sun. However, the distribution of a throughout the entire range which can be observed (up to $2 \cdot 10^4$ astronomical units) has a decreasing nature. Both of these facts can be explained by assuming that all of the pairs which we can observe were formed in the galactic star clusters and then "evaporated" from them. Due to the low "temperature" θ, and to a certain extent due to the high density n of the galactic clusters, the equilibrium number of broad pairs for the clusters is many orders of magnitude higher than the equilibrium number in the area of the sun and is comparable to the number which we actually observe.

Let us now demonstrate that the hypothesis of the formation of binary stars in star clusters quantitatively explains the high percentage of binary stars with half axes in thousands of astronomical units observed. Since the interval of values of a which interests us is near the value of a_0 for the clusters, the approximate method of integration of (2.2) which we used in producing (2.5) cannot be used here. However, we can use the fact that, as numerical calculations show, in the area $0.5 < a/a_0 < 5$, the function $e^{a_0/a}\sqrt{a}$ changes slowly. Therefore, for an approximate estimate in this area we can consider that

$$\int_{a'}^{a''} e^{\frac{a_0}{a}} \sqrt{a}\, da \approx e\sqrt{a_0}\,(a'' - a').$$

In this case

$$\frac{n_{12}}{n_1 n_2} = 4(\pi a_0)^{3/2} \int_{a'}^{a''} e^{\frac{a_0}{a}} \sqrt{a}\, da \approx 4e\pi^{3/2} a_0^2\,(a'' - a'). \tag{2.6}$$

With a density of satellite stars in the cluster of $n_2 = 10$ stars per cubic parsec and $a_0 = 4000$ astronomical units ($v = 10^5$ cm/sec, $m_1 m_2 = m^2_\odot$) the share of binary stars with half axes from 2,000 to 10,000 astronomical units is

$$\frac{n_{12}}{n_1} \approx 10^{-2},$$

which corresponds to the observed facts.

In analyzing the process of formation of broad pairs in star clusters, we must keep in mind that the clusters dissipate and become denser. Pairs with long half axes a should be formed before the mean distance between stars in the cluster becomes less than a. Is this condition actually fulfilled? Suppose the initial density of the cluster is equal to n_0 and /282 $a = \xi n_0^{-1/3}$ where $n_0^{-1/3}$ is the mean distance between stars, ξ is a number less than unity. We are interested in the time after which the density will be such that $a = \xi n_0^{-1/3} = n^{-1/3}$, from which

$$\frac{n}{n_0} = \xi^{-3}.$$

Analysis of the evolution of clusters [7] shows that

$$\frac{n}{n_0} = \left(1 - \frac{t}{t_d}\right)^{-\frac{10}{7}},$$

where t_d is the time of complete dissipation and, consequently, the time which interests us is

$$t = t_d\left(1 - \xi^{\frac{21}{10}}\right).$$

It is necessary that the time of formation of the broad pairs which we are analyzing t_1 be much less than this time. Obviously, this requirement will practically always be fulfilled, since $t_1 \ll t_r \ll t_d$.

In formula (2.6) we did not consider the pairs which were strong under the conditions of the cluster. However, as was estimated at the end of §1, during the time of dissipation of the cluster, pairs will form for which a is an order of magnitude less than a_0. The exponential function in (2.3) for such pairs is quite large, and therefore the total share of the pairs is increased and approaches unity. Data from observations in the area of the sun indicate over 30% multiple stars[1].

Considering that the share of stars of higher multiplicity decreases approximately in a geometric progression, we should expect approximately 10% trinary stars, approximately 3% quadruple stars, approximately 1% quintuple, etc. Since the reduction in distance between stars is accompanied by an increase in the exponential function, a considerable portion of the stars of higher multiplicity should form trapezoid type systems.

Thus, we come to the conclusion that at least a unit share of all the stars in the galaxy came from star clusters. The close connection between this conclusion and the recently stated hypothesis of V. A. Ambartsumyan that the stars of the flat subsystems, and possibly all stars, were formed in associations [8], is obvious.

As was noted above, massive stars form pairs in greater numbers, and for massive stars $2a_0$ may reach values in star clusters equal to the maximum observed half axes of the orbits of star clusters. Therefore, the ascending portion of the curve of distribution of a may be totally absent in clusters where binary stars are formed. On the other hand, after the evaporation of a pair, it finds itself in conditions under which the dissociation of pairs occurs much more frequently than the formation of new pairs. The broader a pair, the more rapidly it will dissociate under the influence of random collisions with other stars. Therefore, the decreasing nature of the distribution of a in the area of the sun may be explained either by the conditions of formation of binary stars in clusters, or by the conditions of their dissociation after "evaporation" from the clusters.

The process of evaporation of broad pairs is not accompanied by their dissociation. The evaporation of pairs, like the evaporation of individual stars, from clusters occurs as a result of the tremendous numbers of very /283
weak energetic forces occurring at median or near median distances between stars. Each such interaction transmits energy, this energy being not only less than the mean energy of the stars, but even less than the energy of the very broadest pairs. Therefore, they do not dissociate, but rather remain in equilibrium number.

The first to evaporate from clusters are single stars of mean mass $\overline{m}$ and pairs of light stars with the same total mass. Gradually, the process encompasses more massive and less massive single stars and, simultaneously,

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[1] Under the conditions in the area of the sun all pairs with $a > 10\text{-}20$ astronomical units are weak, their number exceeds the equilibrium number, and therefore in practice only dissociation will occur here. Pairs with half axes not exceeding a few astronomical units are also strong in the area of the sun, and their number has a tendency to increase.

the corresponding pairs. If the cluster consisted of stars of identical mass, the evaporation of pairs would be strongly retarded.

In conclusion we note that our hypothesis consists of a synthesis of apparently contradictory suggestions concerning the general origin of the components (V. A. Ambartsumyan) and the formation of pairs by the capture mechanism (O. Yu. Shmidt), since we believe that the components were formed in a single association, then joined into pairs by gravitational capture within this association. If the second process, that of formation of binary stars, occurred at an earlier stage, when the first process, that of the formation of stars, had not yet been completed, friction might be very important in the kinetics of captures.

3. Binary stars in rotating clusters. Let us analyze the statistical equilibrium of binary stars in star clusters, rotating as a unit whole, i.e. with nonzero summary moment of the quantity of movement. Let us assume that the angular velocity is independent of the distance from the axis of the cluster. In a frame of reference rotating together with the cluster, the equilibrium distribution will be the ordinary distribution $e^{-u_1/\theta}$, where the expression for energy u_1 includes the "centrifugal energy" [9]:

$$u_1 = \frac{1}{2} m v_1^2 - \frac{1}{2} m (\omega \times r)^2 + u_{\text{pot}}$$

Let us go over to the nonmoving frame of reference: $v_1 = v - \omega \times r$. Then

$$u_1 = \frac{mv^2}{2} - m\mathrm{v}(\omega \times \mathrm{r}) + u_{\text{пот}} = \frac{mv^2}{2} + u_{\text{пот}} - \omega(m\mathrm{r} \times \mathrm{v}) = u - \omega H,$$

where H, as in §2, represents the component of the moment of the quantity of movement of the pair along the axis of rotation of the cluster. Therefore

$$dn_{12} \backsim e^{-\frac{u+\omega H}{\theta}} \, dL\, dG\, dH. \tag{3.1}$$

Introducing the variables a, ϵ and i in place of L, G and H, we have:

$$\frac{\partial(L, G, H)}{\partial(a, \varepsilon, i)} = \frac{\partial L}{\partial a}\frac{\partial G}{\partial \varepsilon}\frac{\partial H}{\partial i} = \frac{\gamma^{3/2} m_1^3 m_2^3}{2(m_1+m_2)^{3/2}} \sqrt{a}\, da\, \varepsilon\, d\varepsilon \sin i\, di,$$

so that

$$dn_{12}(a,\ \varepsilon,\ i) \backsim e^{\frac{\gamma m_1 m_2}{2a\theta}+\frac{\omega G}{\theta}\cos i}\ \varepsilon\sqrt{a}\sin i\, da\, d\varepsilon\, di. \tag{3.2}$$

In these formulas, the movement of the center of inertia of the pair is separated, so that moment G relates only to the internal movement of the pair.

First of all, (3.2) gives us the uneven distribution of orbit inclinations. It is simpler not to analyze the function

$$\frac{dn_{12}}{di} \backsim e^{\frac{\omega G}{\theta}\cos i}\sin i,$$

but rather the density of the poles of the orbits in the celestial sphere /284

$$\frac{dn_{12}}{d\cos i} \backsim e^{\frac{\omega G}{\theta}\cos i}. \tag{3.3}$$

The unevenness of this function appears more sharply, the greater the moment of the movement of the pair G. If all binary stars in the area of the sun came from the same rotating cluster, we can find for them an axis relative to which $dn_{12}/(d \cos i)$ has the form of (3.3). If they came from many clusters, whose axes were symmetrically distributed about the axis of the galaxy, the density of the poles of the orbits of all binaries will also have an axis of symmetry corresponding to the axis of the galaxy.

In order to establish the distribution of eccentricities ϵ, let us integrate (3.1) with respect to H from -G to +G or (3.2) with respect to i from 0 to π:

$$dn_{12}(a,\ \varepsilon) \backsim e^{-\frac{u}{\theta}}\left(e^{\frac{\omega G}{\theta}} - e^{-\frac{\omega G}{\theta}}\right)\frac{\varepsilon\, d\varepsilon\, da}{\sqrt{1-\varepsilon^2}} =$$
$$= e^{\frac{\gamma m_1 m_2}{2a\theta}}\left(e^{\frac{\omega m_1 m_2}{\theta}\sqrt{\frac{\gamma a(1-\varepsilon^2)}{m_1+m_2}}} - e^{-\frac{\omega m_1 m_2}{\theta}\sqrt{\frac{8a(1-\varepsilon^2)}{m_1 m_2}}}\right)\frac{\varepsilon}{\sqrt{1-\varepsilon^2}}\, da\, d\varepsilon. \tag{3.4}$$

This expression contains the correlation between the large half axis and the eccentricity. Therefore, it is possible in principle to use observations of this correlation to estimate the mean angular rate of rotation of the clusters from which the pairs being analyzed originated. For pairs with small moment $G \ll \theta/\omega$,

$$dn_{12}(a, \varepsilon) \sim e^{-\frac{u}{\theta}} \varepsilon \sqrt{a}\, da\, d\varepsilon, \tag{3.4}$$

i.e. the rotation of the clusters, naturally, is not added. For pairs with large moment $G \gg \theta/\omega$, the distribution function decreases with increasing eccentricity, so that the share of pairs with low eccentricity is greater than in the preceding case. Thus, wide pairs should more frequently have low eccentricity than close pairs. Observations show a correlation of the reverse character. The reason for the divergence and the possible role of selection remain unknown.

Finally, rotation decreases the steepness of the drop in the distribution function with increasing half axis for strong pairs and accelerates the increase of this function for weak pairs. Possible explanations as to why this increase is not actually observed are given at the end of §2.

REFERENCES

1. Ambartsumyan, V. A., *Uchenyye Zapis'ki LGU*, v. 22, #4, pp. 19-22, 1938.
2. Ambartsumyan, V. A., *Astronomicheskiy Zhurnal*, 14, pp. 207-219, 1937.
3. Jeans, J., *M. N.*, 79, pp. 408-416, 1919; *Astronomy and Cosmogony*, 1929.
4. Franck, F., and Mizes, R., *Differentsial'nyye i Integral'nyye Uravneniya Matematicheskoy Fiziki* [Differential and Integral Equations of Mathematical Physics], chapter IV, paragraph 1, ONTI Press, 1937.
5. Ambartsumyan, V. A., *Evolyutsiya Zvezd i Astrofizika* [The Evolution of Stars and Astrophysics], Academy of Sciences Armenian SSR Press, 1947.
6. Shayn, G. A., article in *Kurs Astrofiziki i Zvezdnaya Astronomiya* [A Course in Astrophysics and Stellar Astronomy], v. 2, pp. 296-297, ONTI Press, 1936.
7. Gurevich, L. E. and Levin, B. Yu., *Doklady Akademii Nauk SSSR*, 70, pp. 781-784, 1950.
8. Ambartsumyan, V. A., *Astronomicheskiy Zhurnal*, 29, pp. 3-9, 1949.
9. Landau, L. and Livshits, Ye., *Statisticheskaya Fizika* [Statistical Physics], ONTI Press, 1938.

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